Power Series

Monday, April 17, 2023 8:49 AM

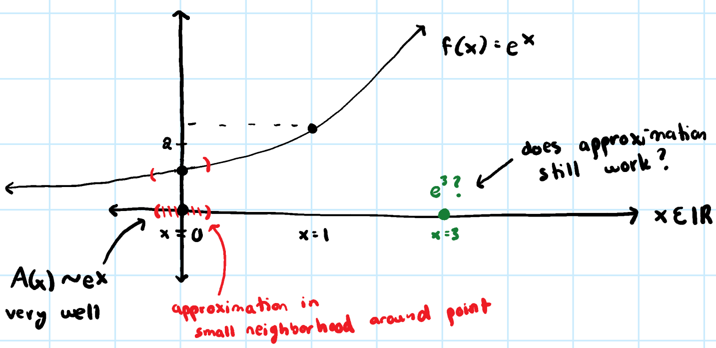
def: the variable x (centered @ 0) is an expression

$$A(x) := \sum_{n=0}^{\infty} a_n x^n \quad \text{where } a_n \in \mathbb{R}$$

$$= a_0 + a_1 x + a_2 x^2 + \dots$$

∞ degree polynomial

why do we do this? → approximate functions @ a point thru polynomials / x values determine convergence



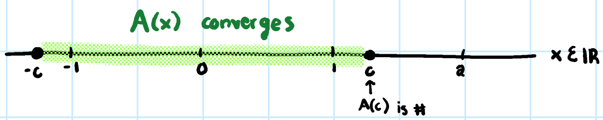
taylor's thm (on wed) @ $x \approx 0$

$$A(x) = \sum_{n \geq 0} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

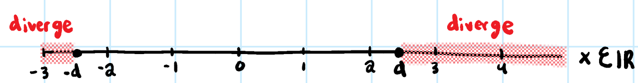
(everything is green / converges)

thm 18: suppose $\sum a_n x^n$ is power series then:

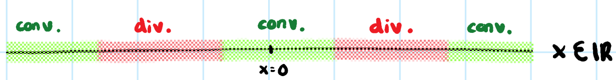
1) if $A(x)$ converges for $x = c$, $c \in \mathbb{R}$, then it converges for $|x| \leq c$ ← converges @ point / would converge @ anything smaller



2) if $A(x)$ diverges @ $x = d$, $d \in \mathbb{R}$, then it diverges for $|x| \geq d$ ← diverges @ point / would diverge @ anything bigger



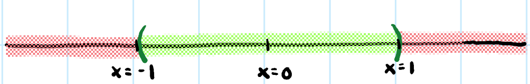
remark: it can not ...



* comparison test *

ex 1) geometric series $(x = r)$: $A(x) = \sum_{n \geq 0} x^n$ $(a_n = 1)$

converges for $|x| < 1$, diverges for $|x| \geq 1$

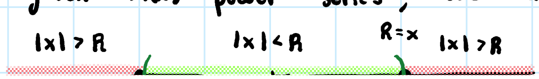


( ) = not included

[ ] = included

corollary of thm 18: given $A(x)$ power series, we have:

1) $\exists R$ such that



← what is R?

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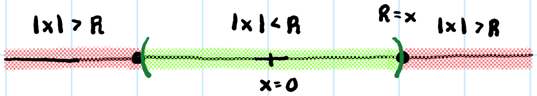


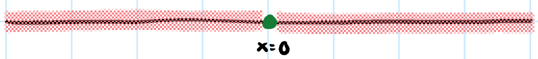
⟵ what is R?

name : radius of convergence

2) all is green:

$R = \infty$

3) all is red except $x = 0$:



$R = 0$

caution! : nothing said for $x = \pm R$ (endpoints) → study those directly (by plugging $x = R$, $x = -R$ and see)